TUNED AMPLIFIERS

• Amplifiers which amplify a specific frequency or narrow band of frequencies are called **tuned amplifiers**.
• Tuned amplifiers are mostly used for the amplification of high or radio frequencies.
• It offers a very high impedance at **resonant frequency** and very small impedance at all other frequencies.

**Advantages of Tuned Amplifiers**

1. Small power loss.
2. High selectivity
3. Smaller collector supply voltage
4. Used in RF amplifiers, Communication receivers, Radar, Television, IF amplifiers
5. Harmonic distortion is very small

**Why not Tuned Circuits for Low Frequency Amplification?**

• **Low frequencies are never single**
• **High values of L and C.**
Classification

Tuned Amplifiers

- Small Signal
  * To amplify low RF signals
  * Power output is low
  * Operated in class A
    - Single tuned
    - Double tuned
    - Staggered tuned

- Large Signal
  * To amplify large RF signals
  * Power output is more
  * Operated in class B, class C or class AB modes.
  * Pushpull configuration used to further reduce harmonic distortion.

Single Tuned Amplifier

- Uses one parallel tuned circuit as the load IZI in each stage and all these tuned circuits in different stages are tuned to the same frequency. To get large Av or Ap, multistage amplifiers are used. But each stage is tuned to the same frequency, one tuned circuit in one stage.

Single tuned amplifiers are further classified as:

- Capacitive coupled
- Transformer coupled or inductive coupled
Single Tuned Capacitive Coupled Amplifier
Single tuned capacitive coupled amplifier:

Single tuned multistage amplifier circuit uses one parallel tuned circuit as a load in each stage with tuned circuits in all stages tuned to the same frequency. Fig. 3.13 shows a typical single tuned amplifier in CE configuration.

As shown in Fig. 3.13 tuned circuit formed by L and C acts as collector load and resonates at frequency of operation. Resistors $R_1$, $R_2$ and $R_E$ along with capacitor $C_E$ provides self bias for the circuit.

![Fig. 3.13 Single tuned capacitive coupled transistor amplifier](image)

Fig. 3.14 Equivalent circuit of single tuned amplifier

The Fig. 3.14 shows the equivalent circuit for single tuned amplifier using hybrid $\pi$ parameters.

As shown in the Fig. 3.14, $R_i$ is the input resistance of the next stage and $R_o$ is the output resistance of the current generator $g_m V_{be}$. The reactances of the bypass capacitor $C_E$ and the coupling capacitors $C_C$ are negligibly small at the operating frequency and hence these elements are neglected in the equivalent circuit shown in the Fig. 3.14.

The equivalent circuit shown in Fig. 3.14 can be simplified by applying Miller's theorem. Fig. 3.15 shows the simplified equivalent circuit for single tuned amplifier.

![Fig. 3.14 Equivalent circuit of single tuned amplifier](image)

Fig. 3.15 Simplified equivalent circuit for single tuned amplifier

Here $C_i$ and $C_{eq}$ represent input and output circuit capacitances, respectively. They can be given as,

$$C_i = C_{be} + C_{bc} (1 - A)$$

where $A$ is the voltage gain of the amplifier. ... (1)

$$C_{eq} = C_{bc} \left(\frac{A-1}{A}\right) + C$$

where $C$ is the tuned circuit capacitance. ... (2)

The $g_{ce}$ is represented as the output resistance of current generator $g_m V_{be}$.

$$g_{ce} = \frac{1}{r_{ce}} = h_{oe} - g_m h_{re} = h_{oe} = \frac{1}{R_o}$$

... (3)
The series RL circuit is represented by its equivalent parallel circuit. The conditions for equivalence are most easily established by equating the admittances of the two circuits shown in Fig. 3.16.

Admittance of the series combination of RL is given as,

\[ Y = \frac{1}{R + j\omega L} \]

Multiplying numerator and denominator by \( R - j\omega L \) we get,

\[ Y = \frac{R - j\omega L}{R^2 + \omega^2 L^2} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} \]

\[ = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega^2 L}{\omega(R^2 + \omega^2 L^2)} \]

\[ = \frac{1}{R_p} + \frac{j\omega L_p}{R_p} \]

where \( R_p = \frac{R^2 + \omega^2 L^2}{R} \) \hspace{1cm} \text{... (4)}

and \( L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} \) \hspace{1cm} \text{... (5)}

**Centre frequency**

The centre frequency or resonant frequency is given as,

\[ f_r = \frac{1}{2\pi \sqrt{L_p C_{eq}}} \] \hspace{1cm} \text{... (6)}

where \( L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} \)

and \( C_{eq} = C_{tr}\left(\frac{A - 1}{A}\right) + C \)

\[ = C_n + C \] \hspace{1cm} \text{... (7)}

Therefore, \( C_{eq} \) is the summation of transistor output capacitance and the tuned circuit capacitance.

**Quality factor Q**

The quality factor \( Q \) of the coil at resonance is given by,

\[ Q_r = \frac{\omega_r L}{R} \] \hspace{1cm} \text{... (8)}

where \( \omega_r \) is the centre frequency or resonant frequency.
This quality factor is also called unloaded Q, but in practice, transistor output resistance and input resistance of next stage act as a load for the tuned circuit. The quality factor including load is called as loaded Q and it can be given as follows:

The Q of the coil is usually large so that $\omega L >> R$ in the frequency range of operation.

From equation (4) we have,

$$R_p = \frac{R^2 + \omega^2 L^2}{R} = R + \frac{\omega^2 L^2}{R}$$

As $\frac{\omega^2 L^2}{R} >> 1$, $R_p = \frac{\omega^2 L^2}{R}$

From equation (5) we have,

$$L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} = \frac{R^2}{\omega^2 L} + L$$

$\therefore \omega L >> R$ 

From equation (9), we can express $R_p$ at resonance as,

$$R_p = \frac{\omega^2 L^2}{R}$$

$$= \omega_r Q_r L \therefore Q_r = \frac{\omega_r L}{R_p}$$

Therefore, $Q_r$ can be expressed in terms of $R_p$ as,

$$Q_r = \frac{R_p}{\omega_r L}$$

The effective quality factor including load can be calculated looking at the simplified equivalent output circuit for single tuned amplifier.

![Fig. 3.17 Simplified output circuit for single tuned amplifier](image)

Effective quality factor $Q_{eff} = \frac{\text{Susceptance of inductance } L \text{ or capacitance } C}{\text{Conductance of shunt resistance } R_t}$

$$Q_{eff} = \frac{R_t}{\omega_r L} \text{ or } \omega_r C_{eq} \frac{R_t}{R_t}$$
**Voltage gain (A_v)**

The voltage gain for single tuned amplifier is given by,

\[
A_v = -g_m \frac{r_{ve}}{r_{bb'} + r_{b'e}} \times \frac{R_t}{1 + 2jQ_{eff} \delta}
\]

where

\[
R_t = \frac{1}{\frac{R_o}{||} + \frac{R_p}{||} + \frac{R_i}{}}
\]

\[
\delta = \text{Fraction variation in the resonant frequency}
\]

\[
\frac{A_v}{A_v (\text{at resonance})} = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times R_t
\]

\[
\therefore \left| \frac{A_v}{A_v (\text{at resonance})} \right| = \frac{1}{\sqrt{1 + (2\delta Q_{eff})^2}}
\]

... (14)

**3 dB bandwidth**

The 3 dB bandwidth of a single tuned amplifier is given by,

\[
\Delta f = \frac{1}{2\pi R_t C_{eq}}
\]

\[
= \frac{\omega_t}{2\pi Q_{eff}}
\]

\[
\therefore Q_{eff} = \omega_t R_t C_{eq}
\]

... (15)

\[
= \frac{f_r}{Q_{eff}}
\]

\[
\therefore \omega_t = 2\pi f_r
\]

... (16)

**DOUBLE TUNED AMPLIFIER:**

The below figure shows double tuned RF amplifier in CE configuration. Here, voltage developed across tuned circuit is coupled inductively to another tuned circuit. Both tuned circuits are tuned to the same frequency.
The double tuned circuit can provide a bandwidth of several percent of the resonant frequency and gives steep sides to the response curve.

**Analysis of double tuned circuits:**

The Fig. 3.19 (a) shows the coupling section of a transformer coupled double tuned amplifier. The Fig. 3.19 (b) shows the equivalent circuit for it. In which transistor is replaced by the current source with its output resistance ($R_o$). The $C_1$ and $L_1$ are the tank circuit components of the primary side. The resistance $R_1$ is the series resistance of the inductance $L_1$. Similarly on the secondary side $L_2$ and $C_2$ represents tank circuit components of the secondary side and $R_2$ represents resistance of the inductance $L_2$. The resistance $R_i$ represents the input resistance of the next stage.

The Fig. 3.19 (c) shows the simplified equivalent circuit for the Fig. 3.19 (b). In simplified equivalent circuit the series and parallel resistances are combined into series elements. Referring equation (9) we have,

$$R_p = \frac{\omega^2 L^2}{R} \text{ i.e. } R = \frac{\omega^2 L^2}{R_o}$$

where $R$ represents series resistance and $R_p$ represents parallel resistance.
Fig. 3.19 Equivalent circuits for double tuned amplifier

Therefore we can write,

\[ R_{11} = \frac{\omega_0^2 L_2^2}{R_o} + R_1 \]

\[ R_{12} = \frac{\omega_0^2 L_2^2}{R_1} + R_2 \]

In the simplified circuit the current source is replaced by voltage source, which is now in series with \( C_1 \). It also shows the effect of mutual inductance on primary and secondary sides.

We know that, \( Q = \frac{\omega L}{R} \)

Therefore, the Q factors of the individual tank circuits are

\[ Q_1 = \frac{\omega_0 L_1}{R_{11}} \quad \text{and} \quad Q_2 = \frac{\omega_0 L_2}{R_{22}} \] \( \ldots (1) \)

Usually, the Q factors for both circuits are kept same. Therefore, \( Q_1 = Q_2 = Q \) and the resonant frequency \( \omega_0^2 = 1/L_1 C_1 = 1/L_2 C_2 \).

Looking at Fig. 3.19 (c), the output voltage can be given as,

\[ V_o = -\frac{j}{\omega_0 C_2} I_2 \] \( \ldots (2) \)

To calculate \( V_o/V_1 \) it is necessary to represent \( I_2 \) in terms of \( V_1 \). For this we have to find the transfer admittance \( Y_T \). Let us consider the circuit shown in Fig. 3.20. For this circuit, the transfer admittance can be given as,
\[ Y_T = \frac{I_2}{V_1} = \frac{I_2}{I_1 Z_{11}} = \frac{A_i}{Z_{11}} \]
\[ = \frac{Z_f}{Z_f^2 - Z_i (Z_o + Z_L)} \]

where
\[ Z_{11} = \frac{V_1}{I_1} = Z_i - \frac{Z_f^2}{Z_o + Z_L} \] and
\[ A_i = \frac{I_2}{I_1} = \frac{-Z_i}{Z_o + Z_L} \]

The simplified equivalent circuit for double tuned amplifier is similar to the circuit shown in Fig. 3.20 with
\[ Z_f = j \omega_r M \]
\[ Z_i = R_{ii} + j \left( \omega L_1 - \frac{1}{\omega C_{1}} \right) \]
\[ Z_o + Z_L = R_{ii} + j \left( \omega L_2 - \frac{1}{\omega C_{2}} \right) \]

The equations for \( Z_f, Z_i \) and \( Z_o + Z_L \) can be further simplified as shown below.
\[ Z_f = j \omega_r M = j \omega_r k \sqrt{\frac{L_1}{L_2}} \]
where, \( k \) is the coefficient of coupling.
Multiplying numerator and denominator by $\omega_r L_1$ for $Z_i$ we get,

\[
Z_i = \frac{R_{11} \omega_r L_1}{\omega_r L_1} + j \omega_r L_1 \left( \frac{Q}{\omega C_1 \omega_r L_1} \right)
\]

\[
= \frac{\omega_r L_1}{Q} + j \omega_r L_1 \left( \frac{Q}{\omega C_1 \omega_r L_1} \right)
\]

\[
\therefore Q = \frac{\omega_r L}{R_{11}} \text{ and } \frac{1}{\omega C_1 \omega_r L_1} = \omega_r C
\]

\[
= \frac{\omega_r L_1}{Q} + j \omega_r L_1 \left( 2 \delta \right)
\]

\[
\therefore \frac{\omega_r}{\omega_r} - \frac{\omega_r}{\omega} = 1 + \delta - (1 - \delta) = 2 \delta
\]

\[
= \frac{\omega_r L_1}{Q} + (1 + j 2 Q \delta)
\]

\[
Z_0 + Z_L = R_{22} + j \left( \frac{\omega L_2 - 1}{\omega C_2} \right)
\]

By doing similar analysis as for $Z_i$ we can write,

\[
Z_0 + Z_L = \frac{\omega_r L_2}{Q} + (1 + j 2 Q \delta)
\]

Then

\[
Y_T = \frac{Z_i}{Z_f - Z_i (Z_0 + Z_L)} = \frac{1}{Z_f - Z_i (Z_0 + Z_L) / Z_f}
\]

\[
Y_T = \frac{1}{j \omega_r k \sqrt{L_1 L_2}} \left[ \frac{\omega_r L_1}{Q} (1 + j 2 Q \delta) \right] \left[ \frac{\omega_r L_2}{Q} (1 + j 2 Q \delta) \right]
\]

\[
Y_T = \frac{kQ^2}{\omega_r \sqrt{L_1 L_2} \left[ 4 Q \delta - j \left( 1 + k^2 Q^2 - 4 Q^2 \delta^2 \right) \right]}
\]

... (3)

Substituting value of $I_2$, i.e. $V_i \times Y_T$ we get,

\[
V_o = \frac{-j}{\omega_r C_2} \frac{g_m V_i}{\omega_r C_1} \left[ \frac{kQ^2}{\omega_r \sqrt{L_1 L_2} \left[ 4 Q \delta - j \left( 1 + k^2 Q^2 - 4 Q^2 \delta^2 \right) \right]} \right]
\]

\[
\therefore \ V_i = j \frac{g_m V_i}{\omega C_1}
\]

\[
A_v = \frac{V_o}{V_i} = g_m \frac{\omega_r L_1 L_2}{\omega_r \sqrt{L_1 L_2} \left[ 4 Q \delta - j \left( 1 + k^2 Q^2 - 4 Q^2 \delta^2 \right) \right]}
\]

\[
\therefore \frac{1}{\omega C} = \omega_r L
\]
\[ \frac{g_m \omega_r \sqrt{L_1 L_2} kQ^2}{4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)} \] ... (4)

Taking the magnitude of equation (4) we have,

\[ |A_v| = g_m \omega_r \sqrt{L_1 L_2} Q \frac{kQ}{\sqrt{1 + k^2Q^2 - 4Q^2\delta^2 + 16Q^2\delta^2}} \] ... (5)

The Fig. 3.21 shows the universal response curve for double tuned amplifier plotted with kQ as a parameter.

The frequency deviation \( \delta \) at which the gain peaks occur can be found by maximizing equation (4), i.e.

\[ 4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2) = 0 \] ... (6)
At \( k^2Q^2 = 1 \), i.e. \( k = \frac{1}{Q} \), \( f_1 = f_2 = f_r \). This condition is known as critical coupling. For values of \( k < 1/Q \), the peak gain is less than maximum gain and the coupling is poor.

At \( k > 1/Q \), the circuit is overcoupled and the response shows the double peak. Such double peak response is useful when more bandwidth is required.

The gain magnitude at peak is given as,
\[
|A_p| = \frac{g_m \omega_0}{\sqrt{L_1 L_2}} \frac{kQ}{2} \quad \ldots (8)
\]

And gain at the dip at \( \delta = 0 \) is given as,
\[
|A_d| = \frac{2kQ}{1 + k^2Q^2} \quad \ldots (9)
\]

The ratio of peak gain and dip gain is denoted as \( \gamma \) and it represents the magnitude of the ripple in the gain curve.
\[
\gamma = \frac{|A_p|}{|A_d|} = \frac{1 + k^2Q^2}{2kQ} \quad \ldots (10)
\]
\[
\gamma = \frac{|A_p|}{|A_d|} = \frac{1 + k^2Q^2}{2kQ} \quad \ldots (10)
\]

Using quadratic simplification and choosing positive sign we get,
\[
kQ = \gamma + \sqrt{\gamma^2 - 1} \quad \ldots (11)
\]

The bandwidth between the frequencies at which the gain is \( |A_d| \) is the useful bandwidth of the double tuned amplifier. It is given as,
\[
\text{BW} = 2\delta' = \sqrt{2} (f_2 - f_1) \quad \ldots (12)
\]

At 3 dB bandwidth,
\[
\gamma = \sqrt{2}
\]
\[
\therefore \quad kQ = \gamma + \sqrt{\gamma^2 + 1} = \sqrt{2} + \sqrt{\sqrt{2}^2 + 1} = 2.414
\]
\[
3 \text{ dB BW} = 2\delta' = \sqrt{2} (f_2 - f_1)
\[
= \sqrt{2} \left[ f_r \left( 1 + \frac{1}{2Q} \sqrt{k^2Q^2 - 1} \right) - f_r \left( 1 - \frac{1}{2Q} \sqrt{k^2Q^2 - 1} \right) \right]
\[
= \sqrt{2} \left[ \frac{f_r}{Q} \sqrt{k^2Q^2 - 1} \right]
\[
= \sqrt{2} \left[ \frac{f_r}{Q} \sqrt{(2.414)^2 - 1} \right] = \frac{3.1 f_r}{Q}
\]
STAGGER TUNED AMPLIFIER:

The double tuned amplifier gives greater 3dB bandwidth having steeper sides and flat top. But alignment of double tuned amplifier is difficult. To overcome this problem two single tuned cascaded amplifiers having certain bandwidth are taken and their resonant frequencies are so adjusted that they are separated by an amount equal to the bandwidth of each stage. Since resonant frequencies are displaced or staggered, they are known as stagger tuned amplifiers. The advantage of stagger tuned amplifier is to have a better flat, wideband characteristics in contrast with a very sharp, rejective, narrow bandwidth characteristics of synchronously tuned circuits (tuned to same resonant frequencies).

Fig. 3.23 shows the relationship of amplification characteristics of individual stages in a staggered pair to the overall amplification of the two stages.

Fig. 3.24 Response of individually tuned and staggered tuned pair
The overall response of the two stage stagger tuned pair is compared in Fig. 3.24 with the corresponding individual single tuned stages having same resonant circuits. Looking at Fig. 3.24, it can be seen that staggering reduces the total amplification of the centre frequency to 0.5 of the peak amplification of the individual stage and at the centre frequency each stage has an amplification that is 0.707 of the peak amplification of the individual stage. Thus the equivalent voltage amplification per stage of the staggered pair is 0.707 times as great as when the same two stages are used without staggering. However, the half power (3 dB) bandwidth of the staggered pair is $\sqrt{2}$ times as great as the half power (3 dB) bandwidth of an individual single tuned stage. Hence the equivalent gain bandwidth product per stage of a staggered tuned pair is $0.707 \times \sqrt{2} = 1.00$ times that of the individual single tuned stages.

The stagger tuned idea can easily be extended to more stages. In case of three stage staggering, the first tuning circuit is tuned to a frequency lower than the centre frequency while the third circuit is tuned to a higher frequency than the centre frequency. The middle tuned circuit is tuned at exactly the centre frequency.

**Analysis**

From equation (14) of section 3.4 we can write the gain of the single tuned amplifier as,

\[
\frac{A_v}{A_v \text{ (at resonance)}} = \frac{1}{1 + 2jQ_{\text{eff}} \delta} = \frac{1}{1 - jX} \quad \text{where } X = 2Q_{\text{eff}} \delta.
\]

Since in stagger tuned amplifiers the two single tuned cascaded amplifiers with separate resonant frequencies are used, we can assume that the one stage is tuned to the frequency $f_r + \delta$ and other stage is tuned to the frequency $f_r - \delta$. Therefore we have,

\[
f_{r1} = f_r + \delta,
\]

and

\[
f_{r2} = f_r - \delta.
\]

According to these tuned frequencies the selectivity functions can be given as,

\[
\frac{A_v}{A_v \text{ (at resonance)}_1} = \frac{1}{1+j(X+1)} \quad \text{and}
\]

\[
\frac{A_v}{A_v \text{ (at resonance)}_2} = \frac{1}{1+j(X-1)}.
\]

The overall gain of these two stages is the product of individual gains of the two stages.
3. Large signal tuned amplifiers:

The output efficiency of an amplifier increases as the operation shifts from class A to class C through class AB and class B. As the output power of a radio transmitter is high and efficiency is prime concern, class B and class C amplifiers are used at the output stages in transmitter.

The operation of class B and class C amplifiers are non-linear since the amplifying elements remain cut-off during a part of the input signal cycle. The non-linearity generates harmonics of the single frequency at the output of the amplifier. In the push-pull arrangement where the bandwidth requirement is not limited, these harmonics can be eliminated or reduced. When a narrow bandwidth is desired, a resonant circuit is employed in class B and class C tuned RF power amplifiers to eliminate the harmonics.
Class B tuned amplifier:

It works with a single transistor by sending half sinusoidal current pulses to the load. The transistor is biased at the edge of the conduction. Even though the input is half sinusoidal, the load voltage is sinusoidal because a high Q RLC tank shunts harmonics to ground. The negative half is delivered by the RLC tank. The Q factor of the tank needs to be large enough to do this. This is analogous to pushing someone on a swing. We only need to push in one direction, and the reactive energy stored will swing the person back in the reverse direction.

Class C tuned amplifier:

The amplifier is said to be class C amplifier, if the Q point and the input signal are selected such that the output signal is obtained for less than a half cycle, for a full input cycle.

Due to such a selection of the Q point, transistor remains active, for less than a half cycle. Hence only that much part is reproduced at the output. For remaining cycle of the input cycle, the transistor remains cut-off and no signal is produced at the output.
From the figure, it is apparent that the total angle during which current flows is less than 180°. This angle is called the conduction angle, $\theta_c$.

The above shows the class C tuned amplifier. Here a parallel resonant circuit acts as a load impedance. As collector current flows for less than half a cycle, the collector current consists of a series of pulses with the harmonics of the input signal. A parallel tuned circuit acting as a load impedance is tuned to the input frequency. Therefore, it filters the harmonic frequencies and produces a sine wave output voltage consisting of fundamental component of the input signal.
To amplify the selective range of frequencies, the resistive load, $R_c$ is replaced by a tuned circuit. The tuned circuit is capable of amplifying a signal over a narrow band of frequencies centered at $f_r$, the amplifiers with such a tuned circuit as a load are known as tuned amplifier.

The above figure shows the tuned parallel LC circuit which resonates at a particular frequency. The resonant frequency and the impedance of tuned circuit is given as,

The response of tuned amplifiers is maximum at resonant frequency and it falls sharply for frequencies below and above the resonant frequency.

In the figure 3 dB bandwidth is denoted as $B$ and 30 dB bandwidth is denoted as $S$. The ratio of 30 dB bandwidth ($S$) to the 3 dB bandwidth ($B$) is known as skirt selectivity.

At resonance, inductive and capacitive effects of tuned circuit cancel each other. As a result, circuit is like resistive and $\cos \phi = 1$ i.e. voltage and current are in phase. For frequencies above resonance circuit is like capacitive and for frequencies below resonance it is like inductive. Since tuned circuit is purely resistive at resonance it can be used as a load for amplifier.
5. Coil losses in tuned amplifiers:

The tuned circuit consists of a coil. Practically, coil is not purely inductive. It consists of few losses and they are represented in the form of leakage resistance in series with the inductor. The total loss of the coil is comprised of copper loss, eddy current loss and hysteresis loss. The copper loss at low frequencies is equivalent to the d.c. resistance of the coil. Copper loss is inversely proportional to the frequency. Therefore, as frequency increase, the copper loss decreases. Eddy current loss in iron and copper coil are due to currents flowing within the copper or core cased by induction. The result of eddy currents is a loss due to heating within the inductors copper or core. Eddy current losses are directly proportional to the frequency. Hysteresis loss is proportional to the area enclosed by the hysteresis loop and increases with frequency. It is function of signal level and increases with frequency.

The total losses in the coil.

QUALITY FACTOR

Quality factor (Q) is important characteristics of an inductor. The Q is the ratio of reactance to resistance and therefore it is unitless. It is the measure of how 'pure' or 'real' an inductor is (i.e. the inductor contains only reactance). The higher the Q of an inductor the fewer losses there are in the inductor. The Q factor also can be defined as the measure of efficiency with which inductor can store the energy. The dissipation factor (D) that can be referred to as the total loss within a component is defined as 1/Q. The Fig. 3.4 shows the quality factor equations for series and parallel circuits and its relation with dissipation factor.

Quality factor for a parallel resonant circuit with loaded and unloaded Q:

\[
Q = \frac{1}{D} = \frac{\omega L_p}{R_p} = \frac{R_p}{\omega L_p}
\]
Loaded and unloaded Q:

Unloaded Q is the ratio of stored energy to dissipated energy in a reactor or resonator. The unloaded Q or $Q_U$ of an inductor or capacitor is $X/R_o$, where $X$ represents the reactance and $R_o$ represents the series resistance. The loaded $Q$ or $Q_L$ of a resonator is determined by how tightly the resonator is coupled to its terminations.

Let us consider the tuned load circuit as shown in the Fig. 3.5. Here, L and C represents tank circuit. The internal circuit losses of inductor are represented by $R_o$ and $R_C$ represents the coupled in load. For this circuit, we can write

$$R_o = \frac{\omega C L}{Q_U} \quad \text{and} \quad R_C = \frac{\omega C L}{Q_L}$$

where $Q_U$ is unloaded Q and $Q_L$ is loaded Q.

![Fig. 3.5 Tuned load circuit](image)

The circuit efficiency for the above tank circuit is given as,

$$\eta = \frac{1 - \frac{R_C}{L (R_C + R_o)}}{12} \times 100\%$$

From above equation it can be easily realized that for high overall power efficiency, the coupled-in load $R_C$ should be large in comparison to the internal circuit losses represented by $R_o$ of the inductor.

The quality factor $Q_L$ determines the 3 dB bandwidth for the resonant circuit. The 3 dB bandwidth for resonant circuit is given as,

$$BW = \frac{f_r}{Q_L}$$

where $f_r$ represents the centre frequency of a resonator and $BW$ represents the bandwidth.

If $Q$ is large, bandwidth is small and circuit will be highly selective. For small $Q$ values bandwidth is high and selectivity of the circuit is lost, as shown in the Fig. 3.6.
Thus in tuned amplifier Q is kept as high as possible to get the better selectivity. Such tuned amplifiers are used in communication or broadcast receivers where it is necessary to amplify only selected band of frequencies.

**Requirements of tuned amplifiers:**

The basic requirements of tuned amplifiers are:

- The amplifier should provide selectivity of resonant frequency over a very narrow band.
- The signal should be amplified equally well at all frequencies in the selected narrow band.
- The tuned circuit should be so mounted that it can be easily tuned. If there are more than one circuit to be tuned, there should be an arrangement to tune all circuits simultaneously.
- The amplifier must provide the simplicity in tuning of the amplifier components to the desired frequency over a considerable range or band of frequencies.
In order to obtain a high overall gain, several identical stages of tuned amplifiers can be used in cascade. The overall gain is the product of the voltage gains of the individual stages. Let us see the effect of cascading of stages on bandwidth.

Consider \( n \) stages of single tuned direct coupled amplifiers connected in cascade. We know that the relative gain of a single tuned amplifier with respect to the gain at resonant frequency \( f_r \) is given from equation (14) of section 3.4.

\[
\left| \frac{A_v}{A_v \text{ (at resonance)}} \right| = \frac{1}{\sqrt{1+(2\delta Q_{eff})^2}}
\]

Therefore, the relative gain of \( n \) stage cascaded amplifier becomes

\[
\left| \frac{A_v}{A_v \text{ (at resonance)}} \right|^n = \left[ \frac{1}{\sqrt{1+(2\delta Q_{eff})^2}} \right]^n = \frac{1}{\left[ 1+(2\delta Q_{eff})^2 \right]^{n/2}}
\]

The 3 dB frequencies for the \( n \) stage cascaded amplifier can be found by equating

\[
\left| \frac{A_v}{A_v \text{ (at resonance)}} \right|^n = \frac{1}{\sqrt{2}}
\]

\[
\left[ 1+(2\delta Q_{eff})^2 \right]^{n/2} = 2^{1/2}
\]

\[
\left[ 1+(2\delta Q_{eff})^2 \right]^n = 2
\]

\[
1+(2\delta Q_{eff})^2 = 2^{1/n}
\]

\[
2\delta Q_{eff} = \pm \sqrt{2^{1/n} - 1}
\]
Substituting for \( \delta \), the fractional frequency variation, i.e.

\[
\delta = \frac{\omega - \omega_r}{\omega_r} = \frac{f - f_r}{f_r}
\]

\[
\therefore \quad 2 \left( \frac{f - f_r}{f_r} \right) Q_{\text{eff}} = \pm \sqrt{2^n - 1}
\]

\[
\therefore \quad 2 (f - f_r) Q_{\text{eff}} = \pm f_r \sqrt{2^n - 1}
\]

\[
\therefore \quad f - f_r = \pm \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1}
\]

Let us assume \( f_1 \) and \( f_2 \) are the lower 3 dB and upper 3 dB frequencies, respectively. Then we have

\[
f_2 - f_r = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1} \quad \text{and similarly,}
\]

\[
f_r - f_1 = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1}
\]

\[
f_2 - f_r = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1} \quad \text{and similarly,}
\]

\[
f_r - f_1 = + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1}
\]

The bandwidth of \( n \) stage identical amplifier is given as,

\[
BW_n = f_2 - f_1 = (f_2 - f_r) + (f_r - f_1)
\]

\[
= \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1} + \frac{f_r}{2 Q_{\text{eff}}} \sqrt{2^n - 1}
\]

\[
= \frac{f_r}{Q_{\text{eff}}} \sqrt{2^n - 1}
\]

\[
= BW_1 \sqrt{2^n - 1}
\]

... (1)

where \( BW_1 \) is the bandwidth of single stage and \( BW_n \) is the bandwidth of \( n \) stages.
Effect of cascading Double tuned amplifier on bandwidth:

When a number of identical double tuned amplifier stages are cascaded in cascade, the overall bandwidth of the system is thereby narrowed and the steepness of the sides of the response is increased, just as when single tuned stages are cascaded. The quantitative relation between the 3 dB bandwidth of $n$ identical stages double tuned amplifiers compared with the bandwidth $\Delta_2$ of such a system can be shown to be

$$n \text{ identical stages double tuned amplifiers} = \Delta_2 \times \left(\frac{1}{2^n} - 1\right)^\frac{1}{4}$$

where $\Delta_2 = 3$ dB bandwidth of single stage double tuned amplifier.
Stability Considerations

- Thermal Effects
- Bias Considerations: Distortion in Audio amplifiers and other types of circuits depends on:
  (i) Input signal level (in mv)
  (ii) Source Resistance
  (iii) Bias Conditions
  (iv) Type of output load and its impedance